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A toy model for coupling accretion disk oscillations to the neutron star spin.

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Abstract. An abstract should be given

Lee, Abramowicz & Kluźniak (2004) demonstrated numerically that rotation of neutron star couples with oscillations of its accretion disk, and excites resonances. No specific coupling was assumed, but magnetic field was suggested as the most likely one. Following this idea, we show (Pétri 2005a, paper I) that if the neutron star is non-axially symmetric and rotating, its gravity may provide the coupling and excite resonances. Here, we return to the original suggestion that the coupling is of a magnetic origin, and demonstrate how does it work in terms of a simple, analytic toy-model.

Key words. Accretion, accretion disks – MHD – Methods: analytical – Relativity – Stars: neutron – X-rays: binaries

1. INTRODUCTION

In some neutron star sources, the observed QPO frequencies obviously depend on the neutron star spin. For example, difference in frequencies of the double peaked QPO in the millisecond pulsar SAX J1808.4-3658 is clearly equal to half of the pulsar spin (Wijnands et al. 2003). This made Kluźniak et al. (2004) to suggest that a resonance is excited by coupling accretion disk oscillation modes to the neutron star spin. The suggestion was fully confirmed by numerical simulations of the coupling (Lee, Abramowicz & Kluźniak, 2004). It was found that a resonant response occurs when the difference between frequencies of the two modes equals to one-half of the spin frequency (as observed in SAX J1808.4-3658 and other "fast rotators"), and when it equals to the spin frequency (as observed in "slow rotators" like XTE J1807-294).

Lee et al. (2004) suggested that the coupling is most likely a magnetic one, but did not specified any concrete mechanism in their numerical simulations, introducing the coupling by a purely formal ansatz. Following their idea, we discussed in terms of a simple analytic toy-model (Pétri 2005a, Paper I) that also (non-axially symmetric) gravitational field of rotating neutron star may excite resonances in accretion disk oscillations.

Another point of view to account for the dichotomy between slow and fast rotators is given by Lamb & Miller (2003) who explain it in the framework of the sonic-point beat frequency model.

In this Research Note, we discuss another toy model that provides the coupling by the neutron star magnetic field. We use the same notation as in Paper I.

2. THE MODEL

In this section, we describe the main features of the model, starting with a simple treatment of the accretion disk, assumed to be made of non interacting charged single particles orbiting in the equatorial plane of the star. Magnetohydrodynamical aspect of the disk such as pressure and current are therefore neglected. Particles evolve in a perfectly spherically symmetric gravitational potential. The asymmetry arises from an eccentric misaligned dipolar magnetic field corotating with the neutron star.

2.1. Eccentric stellar magnetic field

The periodically varying epicyclic frequencies are introduced by adding a rotating asymmetric dipolar magnetic field to the background gravity. Generally, when dealing with an oblique rotator, the location of the magnetic moment $\boldsymbol{\mu}$ generating the dipolar magnetic field coincides with the centre of the neutron star (supposed to be a perfect sphere). In this paper, we lift this assumption and shift the location of the magnetic moment to a point $\boldsymbol{r}_{\rm s}(t) = (r_{\rm s} \neq 0, \varphi_{\rm s} = \Omega_* \, t, z_{\rm s})$ inside the star such that $||\boldsymbol{r}_{\rm s}(t)|| = \sqrt{r_{\rm s}^2 + z_{\rm s}^2} \leq R_*$ where R_* is the stellar radius. We use cylindrical coordinates denoted by (r, φ, z). Nevertheless, the origin of the coordinate system coincides with the centre of the neutron star. Furthermore, in order

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to compute analytically such kind of magnetic field structure, we assume that the star is made of an homogeneous and isotropic matter everywhere with total mass M_* and spinning around its centre at an angular rate $\boldsymbol{\Omega}_* = \Omega_* \, \boldsymbol{e}_{\rm z}$, aligned with the z-axis. The magnetic field induced by the dipolar source is therefore :

$$\boldsymbol{B}(r,\varphi,z,t) = \frac{\mu_0}{4\,\pi} \left[\frac{3\,(\boldsymbol{\mu}\cdot\boldsymbol{R})\,\boldsymbol{R}}{R^5} - \frac{\boldsymbol{\mu}}{R^3} \right] \tag{1}$$

The vector joining the source point $\boldsymbol{r}_{\rm s}(t) = (r_{\rm s}, \varphi_{\rm s}, z_{\rm s})$ to the observer point $\boldsymbol{r} = (r, \varphi, z)$ is :

$$\boldsymbol{R}(t) = \boldsymbol{r} - \boldsymbol{r}_{\rm s}(t) \tag{2}$$

Using the cylindrical frame of reference, the distance between source point and observer is :

$$R^2 = r^2 + r_{\rm s}^2 - 2\,r\,r_{\rm s}\cos\psi + (z - z_{\rm s})^2 \tag{3}$$

where the azimuth in the corotating frame is $\psi = \varphi - \Omega_*\,t$. The magnetic moment anchored in the neutron star, rotates at the stellar speed such that :

$$\boldsymbol{\mu}(t) = \mu\,[\sin\chi\,\{\cos(\Omega_*\,t)\,\boldsymbol{e}_{\rm x} + \sin(\Omega_*\,t)\,\boldsymbol{e}_{\rm y}\} + \cos\chi\,\boldsymbol{e}_{\rm z}] \tag{4}$$

where the obliquity, i.e. the angle between $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}_*$, is denoted by χ. Moreover, each component of the magnetic field can be expressed as follows :

$$B_{\rm r} = \frac{\mu_0}{4\,\pi\,R^3} \left[\frac{3\,(\boldsymbol{\mu}\cdot\boldsymbol{R})\,(r - r_{\rm s}\,\cos\psi)}{R^2} - \mu\,\sin\chi\,\cos\psi \right] \tag{5}$$

$$B_{\varphi} = \frac{\mu_0}{4\,\pi\,R^3} \left[\frac{3\,(\boldsymbol{\mu}\cdot\boldsymbol{R})\,r_{\rm s}\,\sin\psi}{R^2} + \mu\,\sin\chi\,\sin\psi \right] \tag{6}$$

$$B_{\rm z} = \frac{\mu_0}{4\,\pi\,R^3} \left[\frac{3\,(\boldsymbol{\mu}\cdot\boldsymbol{R})\,(z - z_{\rm s})}{R^2} - \mu\,\cos\chi \right] \tag{7}$$

$$\boldsymbol{\mu}\cdot\boldsymbol{R} = \mu\,[\sin\chi\,(r\,\cos\psi - r_{\rm s}) + \cos\chi\,(z - z_{\rm s})] \tag{8}$$

The total linear response of the disk is then the sum of each perturbation corresponding to one particular azimuthal mode m. Because the perturber is inside the star and the disk never reaches the stellar surface, the Fourier coefficients of each component B_i^m $(i = {\rm r}, \varphi, {\rm z})$ never diverge. It is convenient to introduce the Fourier decomposition of the magnetic field component Eq. (5)-(7) in order to describe the response of the test particle. Moreover, because the evaluation of the Fourier coefficients requires to integrate terms containing $\cos(m\,\psi)$ in the integrand, the value of these coefficients decreases rapidly with increasing azimuthal number m. As a result, only low azimuthal modes will influence significantly the evolution of the disk. Keeping only the few first terms in the expansion is sufficient to achieve reasonable accuracy. For discussing the results, we only keep the three first modes, namely, the dipolar, quadrupolar and octupolar moments ($m = 1, 2, 3$ respectively).

2.2. Equation of motion for a charged test particle

All particles evolve in the gravitatomagnetic field imposed by the rotating neutron star. To keep things as simple as possible, their motion is described in the guiding centre approximation. The drift arising from the gyration around the local magnetic field is not an essential feature we want to discuss here. As a consequence, magnetic curvature and gradient as well as gravity drift motions are ignored in this study. Nevertheless, the main characteristic consisting of a periodic variation in the epicyclic frequencies is preserved. The equation of motion then reads :

$$\ddot{\boldsymbol{G}} = \boldsymbol{g} + \frac{q_{\rm e}}{m_{\rm e}}\,\dot{\boldsymbol{G}} \wedge \boldsymbol{B} \tag{9}$$

where $\boldsymbol{G}$ is the location of the guiding centre and the dot means time derivative d/dt. The gravitational field of the star M_* is denoted by $\boldsymbol{g} = \boldsymbol{\nabla}(G\,M_*/\sqrt{r^2+z^2})$. The mass and the charge of the particle are denoted respectively by $m_{\rm e}$ and $q_{\rm e}$. If for instance the magnetic gradient drift is taken into account, the term $-(\mu_{\rm e}/2\,m_{\rm e}\,B)\,\boldsymbol{\nabla}B^2$ should be added to the right hand side of Eq. (9) where $\mu_{\rm e}$ is an adiabatic invariant, namely the magnetic moment of the test particle gyrating along the local field line. This would introduce another modulation of the epicyclic frequencies which is already included in the Lorentz force $q_{\rm e}\,\dot{\boldsymbol{G}} \wedge \boldsymbol{B}$. Thus, the physical behaviour is not changed by neglecting the drift motion of the guiding centre $\boldsymbol{G}$. Expressed in cylindrical coordinates, Eq. (9) develops into

$$\ddot{r} - r\,\dot{\varphi}^2 = g_{\rm r} + \frac{q_{\rm e}}{m_{\rm e}}\,(r\,\dot{\varphi}\,B_{\rm z} - \dot{z}\,B_{\varphi}) \tag{10}$$

$$2\,\dot{r}\,\dot{\varphi} + r\,\ddot{\varphi} = \frac{q_{\rm e}}{m_{\rm e}}\,(\dot{z}\,B_{\rm r} - \dot{r}\,B_{\rm z}) \tag{11}$$

$$\ddot{z} = g_{\rm z} + \frac{q_{\rm e}}{m_{\rm e}}\,(\dot{r}\,B_{\varphi} - r\,\dot{\varphi}\,B_{\rm r}) \tag{12}$$

The magnetic field $\boldsymbol{B}$ is assumed to be weak enough for the flow to remain essentially hydrodynamical (weakly magnetized thin disk approximation). We therefore treat $\boldsymbol{B}$ as a perturbation of order $\varepsilon \ll 1$. The perturbations induced in the flow are of the same order of magnitude than $\boldsymbol{B}$, i.e. of order ε. The perturbed orbit and velocity of the test particle in the radial and vertical direction are also of order ε, $\{\dot{r}, \dot{z}\} = {\rm O}(\varepsilon)$ whereas the azimuth varies as $\dot{\varphi} \approx \Omega_{\rm k} \approx \sqrt{G\,M_*/r_0^3} + {\rm O}(\varepsilon)$, the Keplerian orbital frequency at the radius of the orbit r_0. In the equations of motion (10), (11) and (12), terms such as $\{\dot{r}, \dot{z}\} \times B_i$ with $i = \{{\rm r}, \varphi, {\rm z}\}$ are second order ${\rm O}(\varepsilon^2)$ and we neglect them. According to this simplification, the right hand side of Eq. (11) vanishes. Eq. (11) states the conservation of angular momentum of the particle and integrates into $L = m\,r^2\,\dot{\varphi} = {\rm const}$ where L is the angular momentum of the particle. This is an obvious integral of motion for this problem (to the aforementioned approximation).

However, we are only interested in the vertical motion experienced by the test particles in response to the perturbed magnetic field. Indeed, the response of an accretion disc to an inclined rotating magnetic dipole has been studied by Terquem & Papaloizou (2000). They showed that

the vertical displacement (i.e. the warping) is the dominant effect in the thin disc while the horizontal perturbations can be neglected. Perturbing (12) and developing to first order in the perturbation around the equilibrium Keplerian orbit (contained in the equatorial plane) defined by $(r_0, \varphi_0 = \Omega_{\rm k}\, t, z_0 = 0)$, the vertical motion reads :

$$\ddot{z} = g_{\rm z} - \frac{q_{\rm e}}{m_{\rm e}}\, r_0\, \Omega_{\rm k}\, B_{\rm r} \tag{13}$$

To avoid mathematical irrelevant complications, we assume the rotator to be "aligned" with the star in the sense that $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}_*$ are parallel ($\chi = 0$). Therefore the radial component of the magnetic field reads :

$$B_{\rm r} = \frac{3\,\mu_0\,\mu}{4\,\pi}\,\frac{(z - z_{\rm s})\,(r - r_{\rm s}\,\cos\psi)}{R^5} \tag{14}$$

By developing in a Fourier series, we obtain :

$$B_{\rm r} = (z - z_{\rm s})\,\sum_{m=0}^{+\infty} B_{\rm r}^m(r,z)\,\cos{(m\,\psi)} \tag{15}$$

where the Fourier coefficients are given by :

$$B_{\rm r}^m(r,z) = \frac{3\,\mu_0\,\mu}{4\,\pi}\,\frac{2-\delta_m^0}{2\,\pi}\,\int_0^{2\pi}\frac{r - r_{\rm s}\,\cos\psi}{R^5}\,\cos{(m\,\psi)}\,d\psi \tag{16}$$

where δ_m^0 is the Kronecker symbol. Note that $B_{\rm r}^m$ do not have the dimension of a magnetic field because of their definition Eq. (15). These coefficients, which are function of the space position (r,z) decrease with increasing azimuthal number m (for (r,z) fixed). Keeping the first few coefficients is sufficient to achieve a reasonable accuracy (we retain the first three). Putting the expansion Eq. (15) into the vertical equation of motion Eq. (13), we get the fundamental equation to describe vertical forced oscillations of a test particle as follows :

$$\left[\Omega_{\rm k}^2 + \frac{q_{\rm e}}{m_{\rm e}}\,r_0\,\Omega_{\rm k}\sum_{m=0}^{+\infty}B_r^m(r_0,z_0)\,\cos\{m(\Omega_{\rm k}-\Omega_*)\,t\}\right]z$$
$$+\ddot{z} = \frac{q_{\rm e}}{m_{\rm e}}\,r_0\,\Omega_{\rm k}\,z_{\rm s}\sum_{m=0}^{+\infty}B_r^m(r_0,z_0)\,\cos\{m\,(\Omega_{\rm k}-\Omega_*)\,t\} \tag{17}$$

Note that the Fourier coefficients $B_r^m(r_0,z_0)$ in this last equation are evaluated at the location of the unperturbed orbit and do no longer depend on the perturbed position (r,z). To the lowest order of the expansion, this approximation is justified. We recognize a Hill equation (periodic variation of the eigenfrequency of the system on the left hand side) with a periodic driving force (on the right hand side).

2.3. Resonance conditions

This equation is very similar to the one obtained in the case where solely gravity perturbation exists. The discussion is therefore exactly the same as in paper I. Here we recall the main results, adapted to the magnetic configuration. Eq. (17) describes an harmonic oscillator with periodically varying eigenfrequency which is also excited by a driven force. It is well known that some resonances will therefore occurs in this system. Namely, we expect three kind of resonances corresponding to :

- a *corotation resonance* at the radius where the angular velocity of the test particle equals the rotation speed of the magnetic structure (which is the equal to the stellar rotation rate). Corotation is only possible for prograde motion. The resonance condition determining the corotating radius is simply $\Omega_{\rm k} = \Omega_*$;
- a *driven resonance* at the radius where the vertical epicyclic frequency equals the frequency of each mode of the magnetic perturbation as seen in the locally corotating frame. The resonance condition is $m\,|\Omega_* - \Omega_{\rm k}| = \kappa_{\rm z}$;
- a *parametric resonance* related to the time-varying vertical epicyclic frequency, (Hill equation). The rotation of the magnetosphere induces a sinusoidally variation of the vertical epicyclic frequency leading to the well known Mathieu's equation for a given azimuthal mode m. The resonance condition is derived as followed :

$$m\,|\Omega_* - \Omega_{\rm k}| = 2\,\frac{\kappa_{\rm z}}{n} \tag{18}$$

where $n \geq 1$ is a natural integer. Note that the driven resonance is a special case of the the parametric resonance for $n = 2$. However, their growth rate differ by the timescale of the amplitude magnification. Driving causes a linear growth in time while parametric resonance causes an exponential growth. We also rewrite the vertical epicyclic frequency as $\kappa_{\rm z}$ instead of $\Omega_{\rm k}$ in order to apply the results to a more general case which could include stronger magnetic fields or general relativistic effects.

Consequently, the resonance conditions for the magnetized rotator are exactly the same as for the unmagnetized rotator of paper I, as long as the oscillations remain in the linear regime (i.e. in the thin disk approximation for which the vertical motion of the particle remains small with respect to the radius of the unperturbed orbit). For more details and a discussion on these results, we refer the reader to paper I.

The dichotomy between the QPOs in fast and slow rotators has been explained by Lee, Abramowicz & Kluźniak (2004) as a consequence of the fact that the coupling between the neutron star spin and the modes of accretion disk oscillations excites possible resonances at different locations in the disk, either very close to the star, or far away from it. While we agree with this results, we point out that (as found in Paper I) the spectrum of modes that *could be in a resonance* is more complex than that consider by Lee et al. Their discussion was concentrated on the 3:2 resonance that occurs between the radial and vertical epicyclic modes *only in strong gravity.* We identified possible forced resonances between different modes, that may occur in both strong and weak gravity. Our analysis

was done in a linear regime, so, at the moment, we are only able to say that such resonances may in principle exist[1].

3. CONCLUSION

The toy-model discussed in this *Research Note* illustrates the idea (Kluźniak et al., 2004; Lee et al. 2004; Pétri 2005a) that the double peak QPOs in neutron stars sources may be due to coupling rotation of neutron star sources to modes in accretion disk oscillations, and exciting resonances.

The model is physically very specific. It explicitly shows how the rotating magnetic field of the neutron stars could couple with the disk dynamics and oscillations. It confirms general discussion and numerical results of Kluźniak et al., (2004) and Lee et al. (2004), in particular the important one that the strongest resonant response occurs when the difference between frequencies of the two modes equals to one-half of the spin frequency (as observed in SAX J1808.4-3658 and other "fast rotators"), and when it equals to the spin frequency (as observed in "slow rotators" like XTE J1807-294).

When the MHD nature of the flow is taken into account, QPOs can be explained by a mechanism similar to those exposed here (Pétri 2005b). However, in an accreting system in which the neutron star is an oblique rotator, we expect a perturbation in the magnetic field to the same order of magnitude than the unperturbed one. Therefore, the linear analysis developed in this paper has to be extended to oscillations having non negligible amplitude compared to the stationary state. Nonlinear oscillations therefore arise naturally in the magnetized accretion disk. Abramowicz et al. (2003) showed that the nonlinear resonance for the geodesic motion of a test particle can lead to the 3:2 ratio for the two main resonances. Nevertheless, an extension to 3D MHD flow in curved spacetime is required to make quantitative accurate predictions of the peak frequencies variation correlated with the accretion rate for instance.

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[1] I was informed by M. Abramowicz, the referee, that unpublished numerical results of Lee et al. fully confirms existence of these additional, weak gravity's, resonances.